Oceanology International
23-25 OCT 2018, Qingdao, China

Dynamic Analysis of the System of the Moored-spherical-buoy

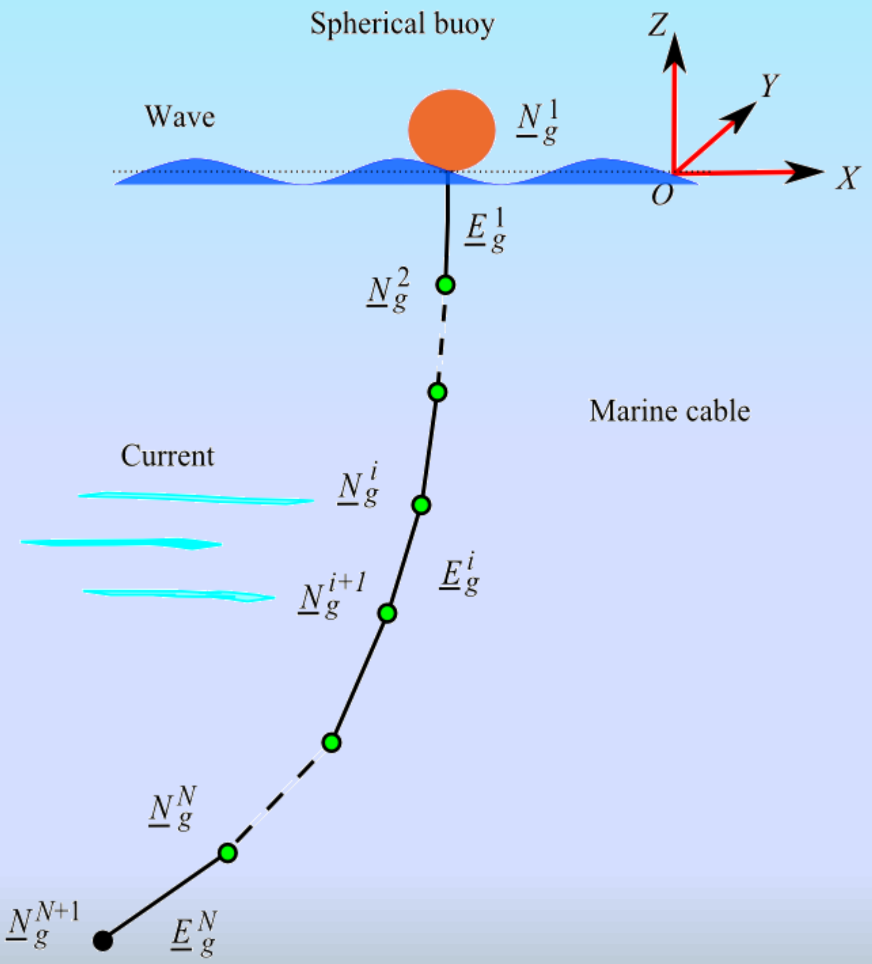
Prof. Xiangqian Zhu

Department of Mechanical Engineering,
Shandong University

Modeling of Cable
1. Background
2. Formulation
3. Element reference frame
4. Advantages of new ERF

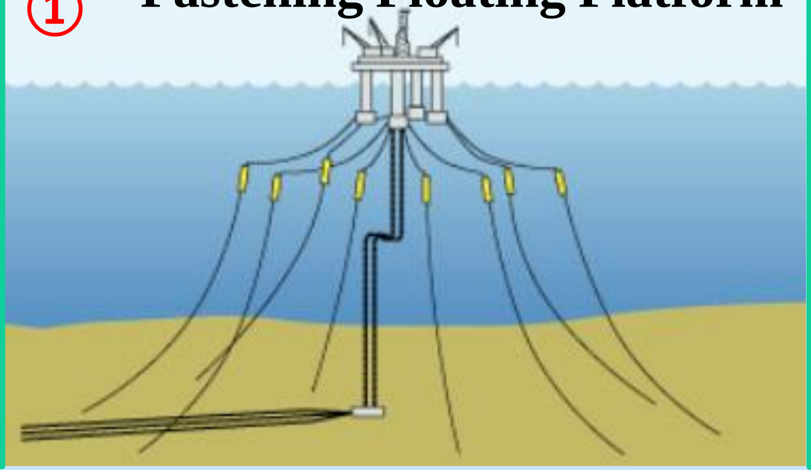
Modeling of Spherical Buoy with Cable
1. Geometrical modeling
2. Formulations of loads
3. Spherical buoy with cables

Summary



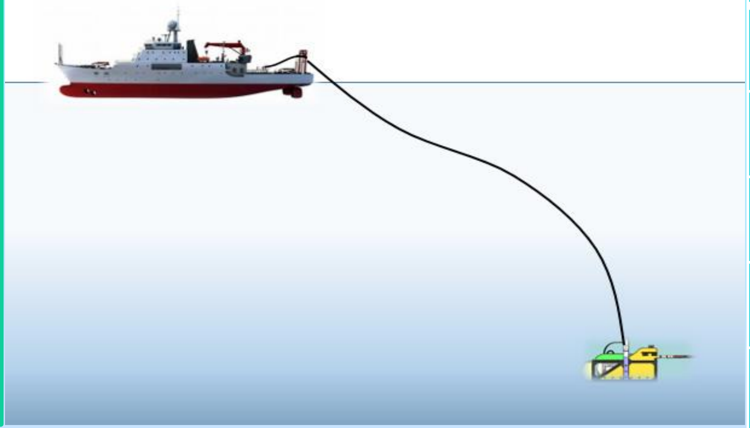
Quasi-static Analysis

① Fastening Floating Platform



Dynamic Analysis

② Towing Marine Vehicles



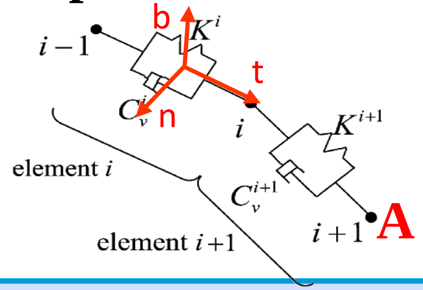
No.	①	②
Tension	O	O
Strain	X	O
Damping	X	O
Hydrodyn	X	O
Efficiency	●●	●
Accuracy	●	●●

Development New Technology

- Small Structural
- Composite Materials
- Accuracy Requirement
- Computer Technology

Dynamics of cables should be considered in analyzing floating structures with mooring cables !

Lumped mass model

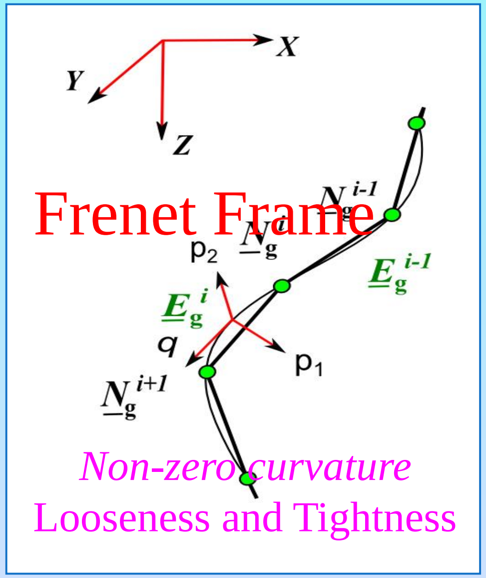


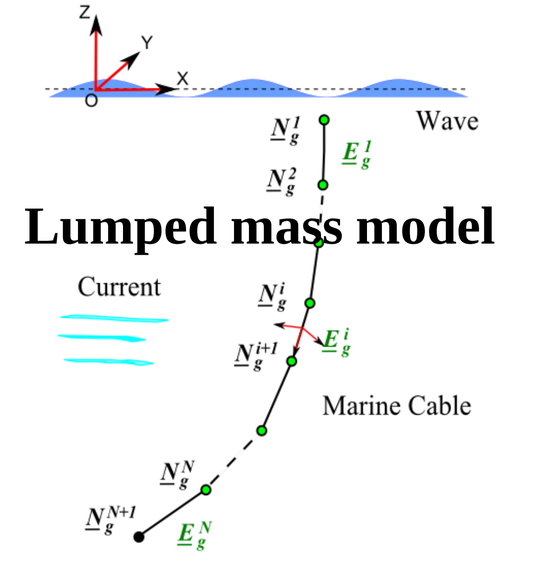
Finite difference model

$$\int_{t_1}^{t_2}\int_0^L \rho A(\vec{v}\cdot\delta\vec{v})\,ds\,dt$$

B

A	B
T	T & S
ODE	PDE
Low- order	High-order
Robust	Sensitive





$$\begin{bmatrix} \cos\theta^i & \sin\theta^i \sin\phi^i & \sin\theta^i \cos\phi^i \\ 0 & \cos\phi^i & -\sin\phi^i \\ -\sin\theta^i & \cos\theta^i \sin\phi^i & \cos\theta^i \cos\phi^i \end{bmatrix}$$

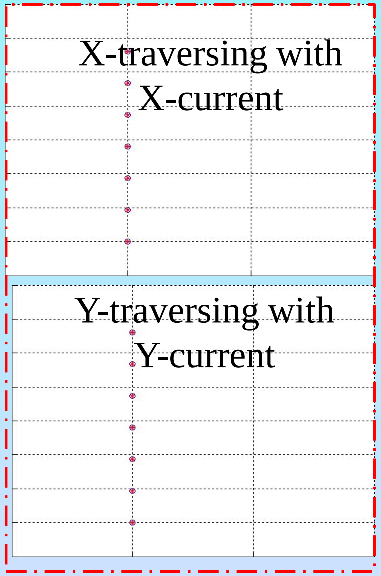
$$\theta^i = \operatorname{atan2}\left(N_g^{i+1,1} - N_g^{i,1},\ N_g^{i+1,3} - N_g^{i,3}\right)$$

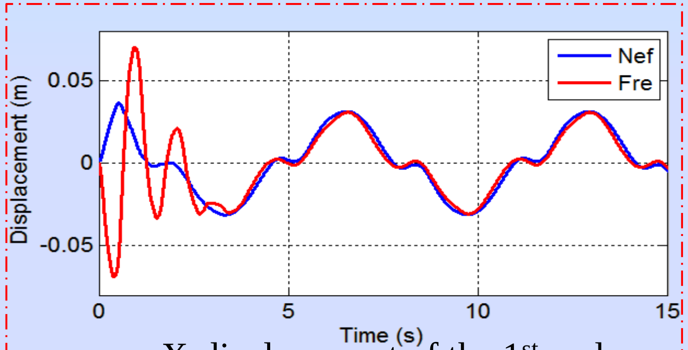
$$\phi^i = \operatorname{atan2}\left(-(N_g^{i+1,2} - N_g^{i,2}),\ \frac{N_g^{i+1,3} - N_g^{i,3}}{\cos\theta^i}\right), \text{if } \cos\theta^i > \sin\theta^i$$

$$\phi^i = \operatorname{atan2}\left(-(N_g^{i+1,2} - N_g^{i,2}),\ \frac{N_g^{i+1,1} - N_g^{i,1}}{\sin\theta^i}\right), \text{if } \cos\theta^i < \sin\theta^i$$

Euler Angles

Gimbal lock
Calculation of rotation angles





X-displacement of the 1st node

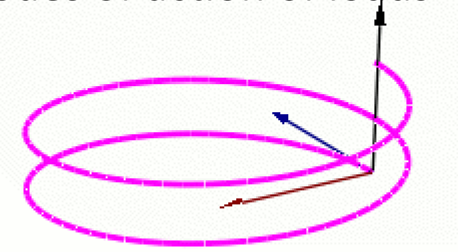
A new Element frame

1. Deal with singular problems
2. Express loads efficiently

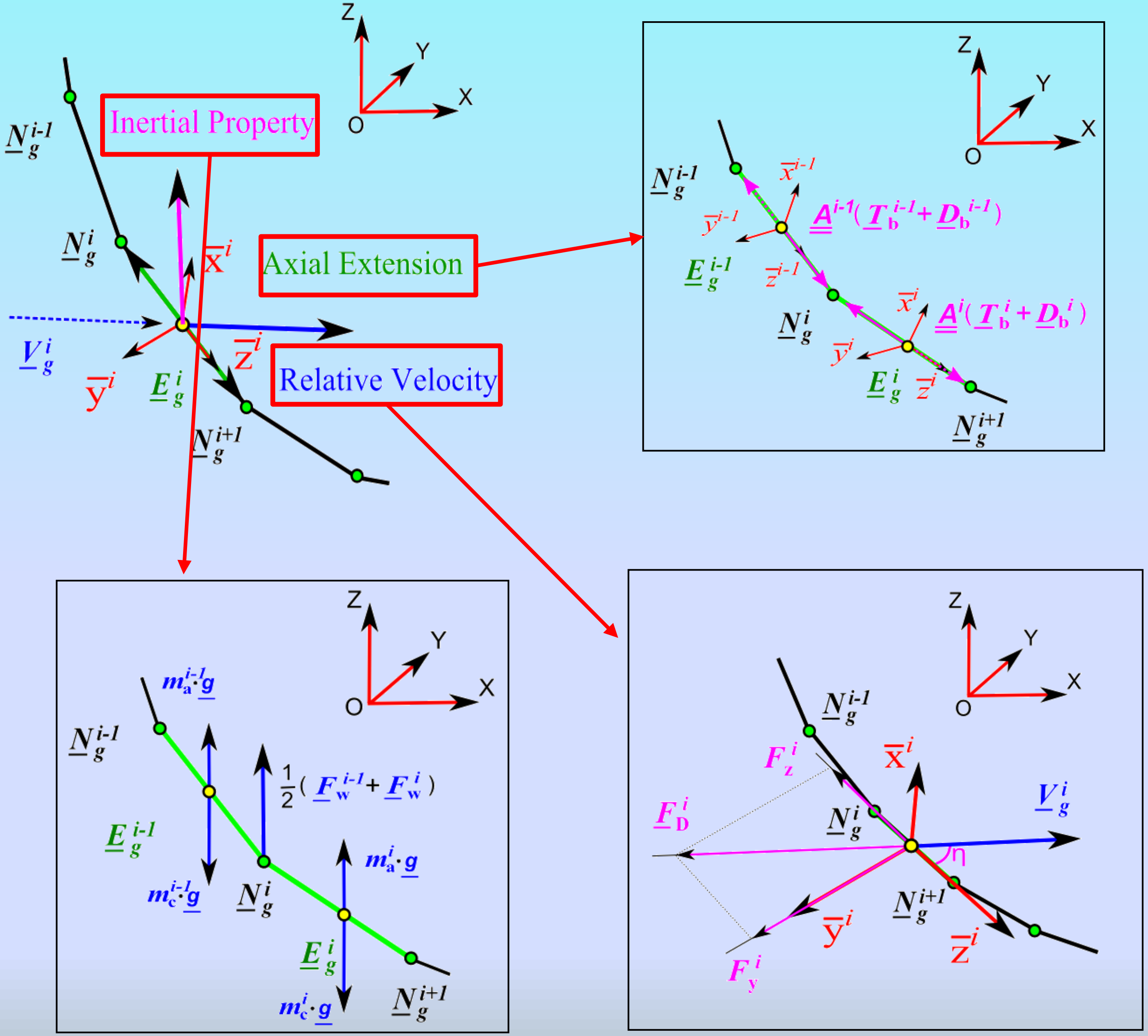
To consider the influence of the dynamics of cables on the system of floating structures with mooring cables, the cable modeling is established based on a new element frame through which not only the cable modeling is competent to model the situations where the singular problems are generated by the Euler angles and Frenet frame, but also the hydrodynamic loads acting on cables are expressed efficiently.

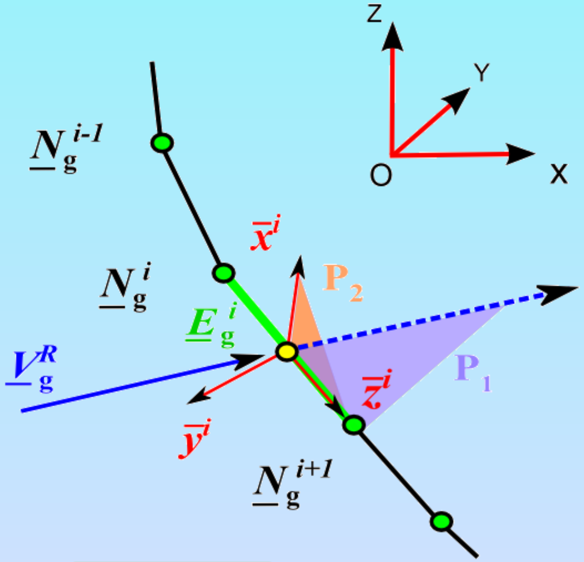
"Shape" ---static

"Environment"
Modes of action of loads



"Loads" ---dynamics





$$E_g^i = N_g^{i+1} - N_g^i$$

$$V_g^i = \frac{\dot{N}_g^{i+1} + \dot{N}_g^i}{2}$$

$$V_g^R = V_g^f - V_g^i$$

$$\bar{z}^i = \frac{E_g^i}{\| E_g^i \|}$$

$$\bar{x}^i = \frac{\bar{z}^i \times V_g^R}{\| \bar{z}^i \times V_g^R \|}$$

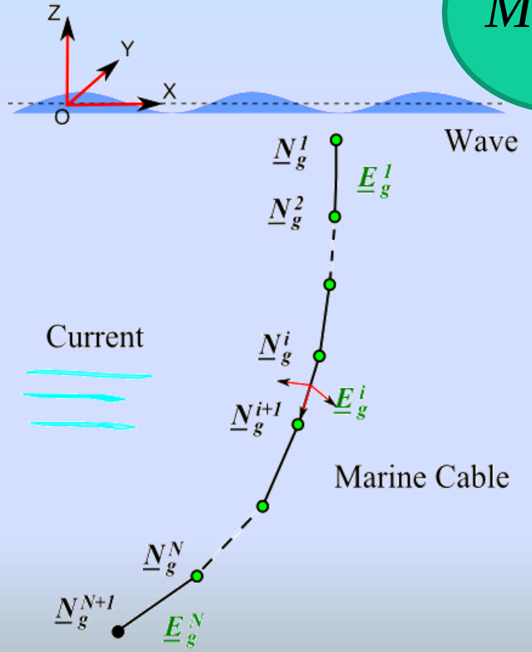
$$\bar{y}^i = \bar{z}^i \times \bar{x}^i$$

$$A^i = [\bar{x}^i, \bar{y}^i, \bar{z}^i]$$

$$A^i = \begin{bmatrix} \cos(u_\xi,u_x) & \cos(u_\eta,u_x) & \cos(u_\zeta,u_x) \\ \cos(u_\xi,u_y) & \cos(u_\eta,u_y) & \cos(u_\zeta,u_y) \\ \cos(u_\xi,u_z) & \cos(u_\eta,u_z) & \cos(u_\zeta,u_z) \end{bmatrix}$$

Governing Equation for Modeling of Cable

Loads acting on i^{th} Node



$$M_I^i \ddot{N}_g^i = \underline{\underline{A}}^i \left(\underline{T}_b^i + \underline{D}_b^i + \frac{1}{2} \underline{F}_D^i \right) + \frac{1}{2} \underline{F}_W^i - \underline{\underline{A}}^{i-1} \left(\underline{T}_b^{i-1} + \underline{D}_b^{i-1} - \frac{1}{2} \underline{F}_D^{i-1} \right) + \frac{1}{2} \underline{F}_W^{i-1} + \underline{F}_{ext}^i$$

Relate ***ERF*** with ***IRF***

by Axial extension W.R.T. ***ERF***

Loads acting on ***i^{th} Element***

Loads acting on ***(i-1)th Element***

by Relative velocity W.R.T. ***ERF***

Loads acting on ***i^{th} Node.***

Parameters	Magnitude	Unit
Diameter	0.03	m
Density	7800	kg/m^3
Elastic modulus	2.0e11	N/m
Damping coefficient	1.0e4	Ns/m
Normal drag coefficient	1	
Tangential drag coefficient	0.01	
Added mass coefficient	1	
Position of 1st node	[10; 0; -10]	m
Position of 11th node	[0; 0; -20]	m

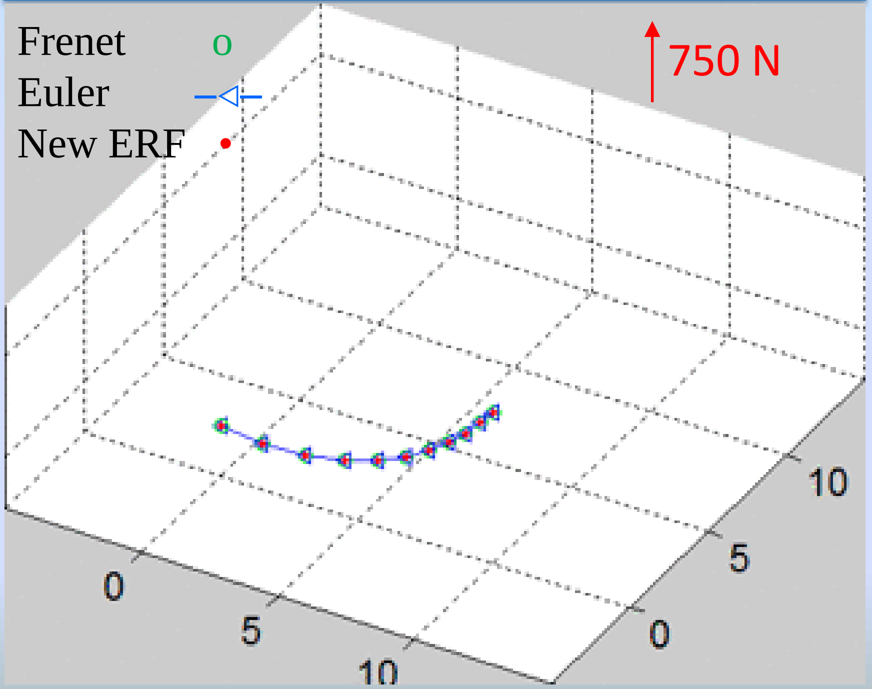
Parameters	Magnitude	Unit
X-directional wave amplitude	1.2	m
X-directional wave period	8	S
Velocity of current	[0; 1; 0]	m/s
Density of fluid	1025	kg/m^3
Stiffness coefficient	8.70e7	N/m
Damping coefficient	3.30e6	Ns/m

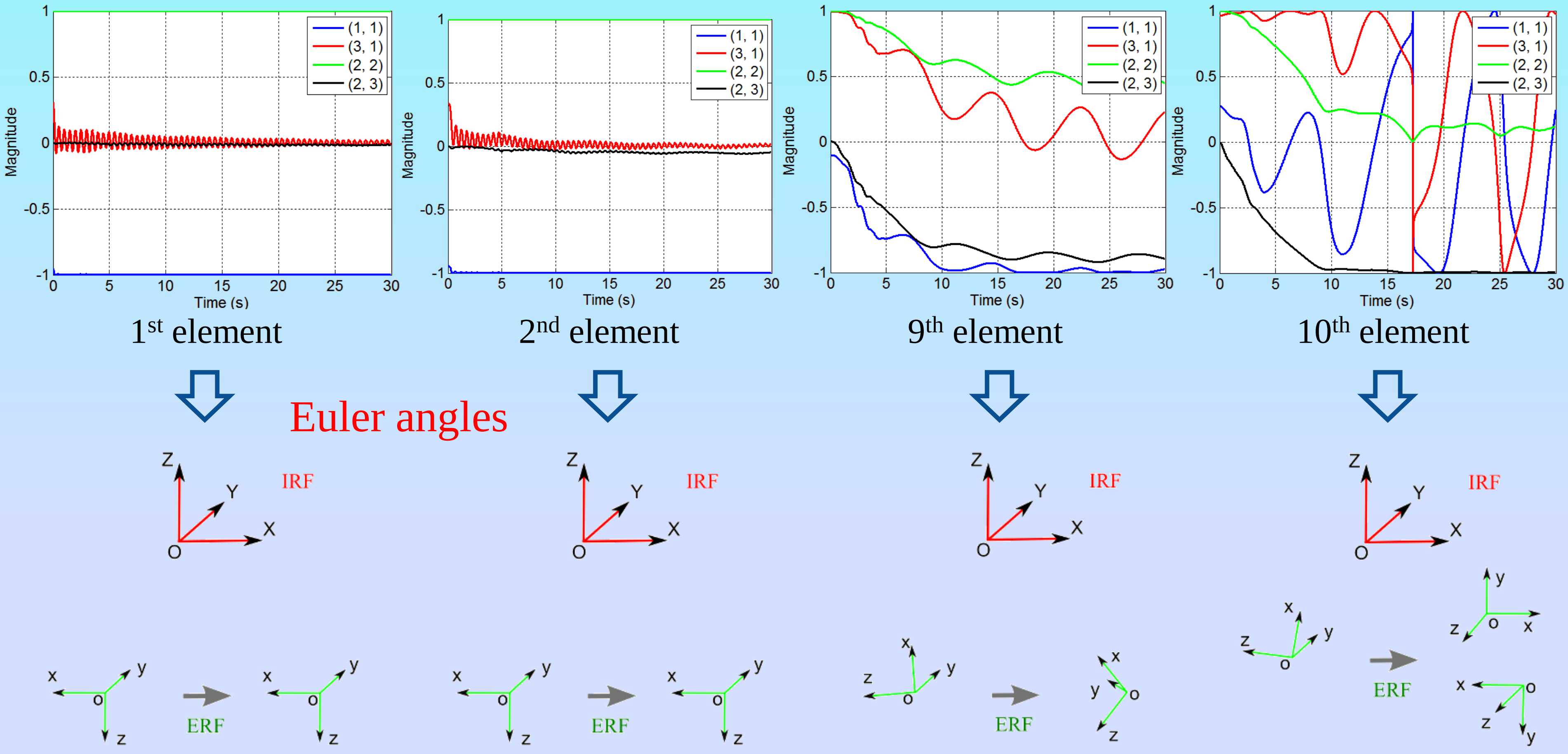
$$\underline{\underline{A}}^i = \begin{bmatrix} \cos\theta^i & \sin\theta^i \sin\phi^i & \sin\theta^i \cos\phi^i \\ 0 & \cos\phi^i & -\sin\phi^i \\ -\sin\theta^i & \cos\theta^i \sin\phi^i & \cos\theta^i \cos\phi^i \end{bmatrix}$$

Euler angles (XYZ) set
Set Z is zero

$$\underline{\underline{A}}^i = \begin{bmatrix} \cos(u_\xi, u_x) & \cos(u_\eta, u_x) & \cos(u_\zeta, u_x) \\ \cos(u_\xi, u_y) & \cos(u_\eta, u_y) & \cos(u_\zeta, u_y) \\ \cos(u_\xi, u_z) & \cos(u_\eta, u_z) & \cos(u_\zeta, u_z) \end{bmatrix}$$

Frenet frame & New Element frame



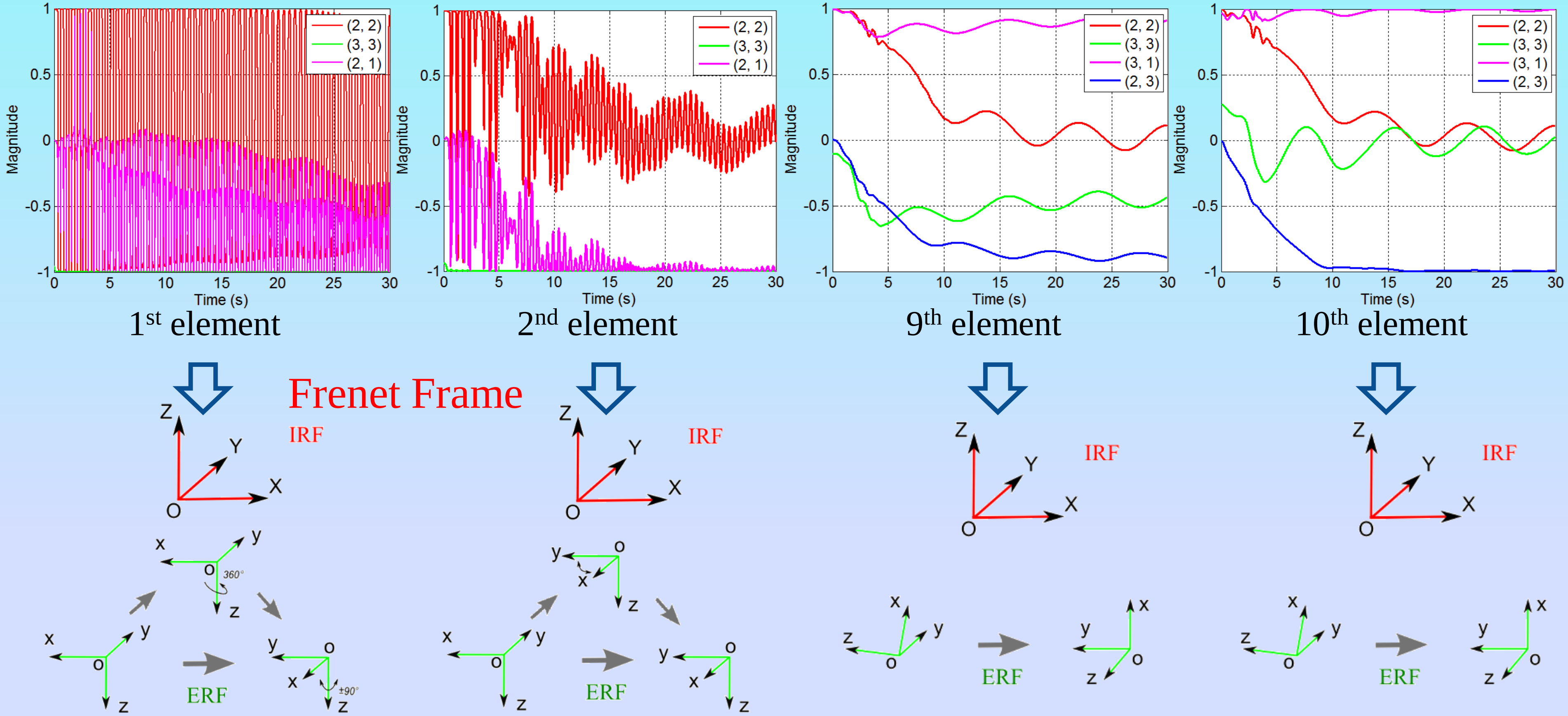


Therefore, the following characteristics of the Euler angles can be concluded from above results:

(1) rotation angles of one element are only related to the direction of the z-axis that indicates the orientation of the said element;

(2) the RTM of one element is merely identified by the orientation of the said element.

CABLE MODELING---ERF



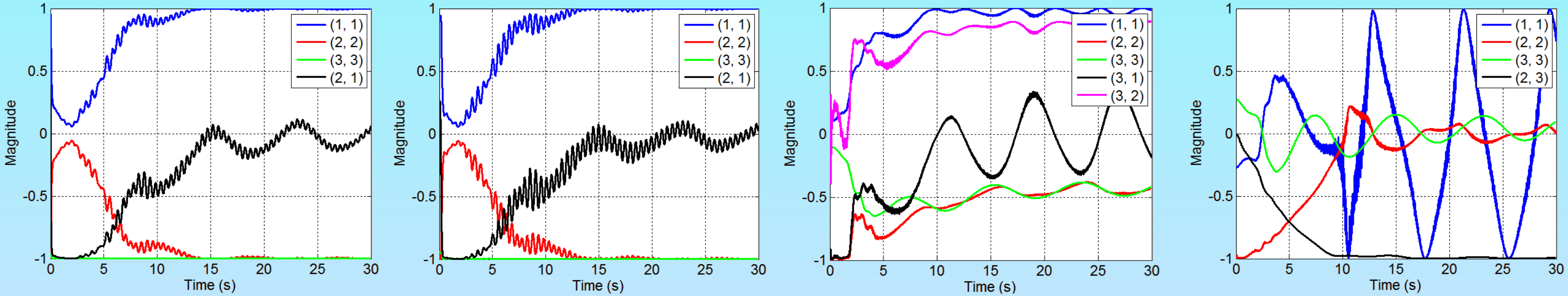
Therefore, characteristics of the Frenet frame are:

(1) the components of the RTM change to ensure that the z-axis is tangential to the cable and that the x-axis is normal to the cable;

(2) the ERF for one element is defined by the said element and two adjacent elements together; and

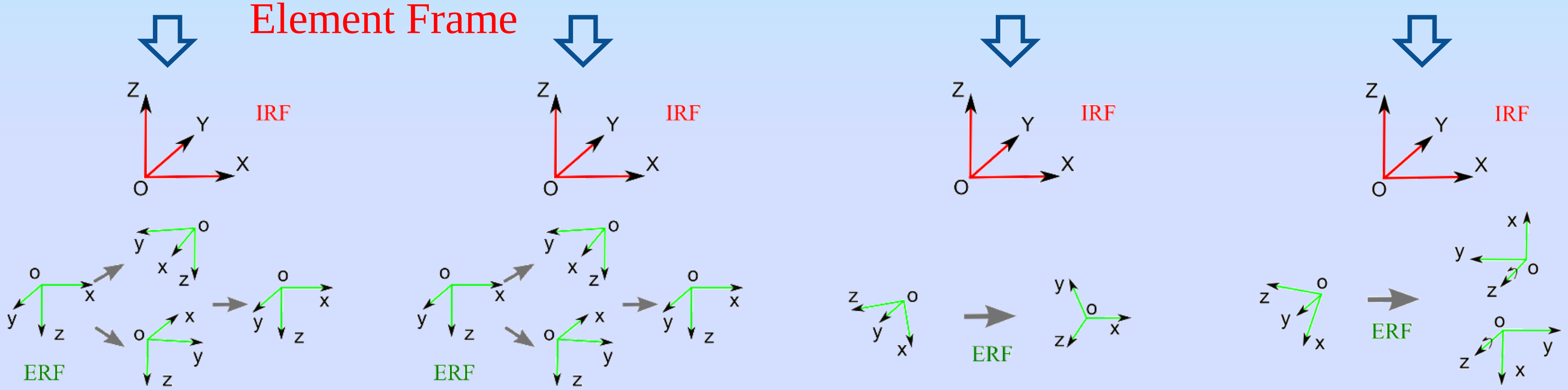
(3) the normal vector is easily undefined when the second derivative of the spline function of the cable becomes very small.

CABLE MODELING---ERF



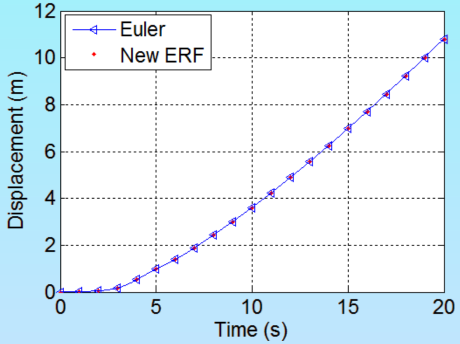
1st element 2nd element 9th element 10th element

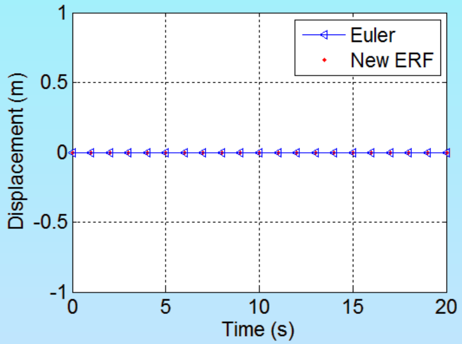
New
Element Frame

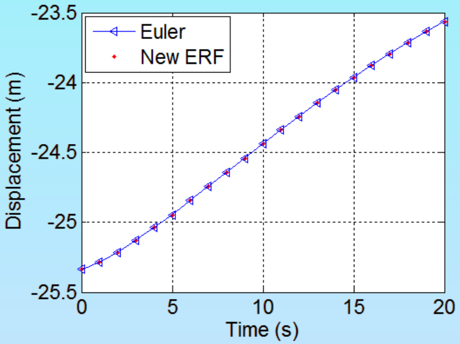


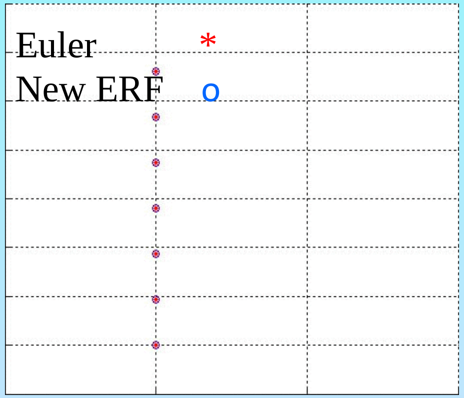
The new ERF is generated on a basis of the following assumptions:
(1) the vector of the relative velocity is non-zeros; and
(2) the vector of the relative velocity is non-collinear with the vector of the element orientation.

Move **X**-directionally with **X**-directional current

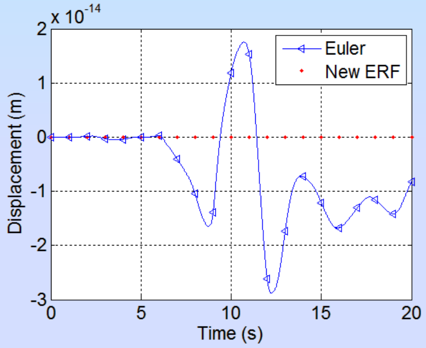


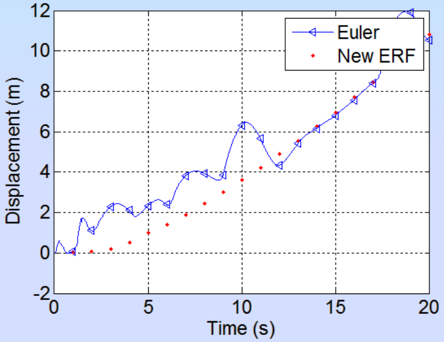


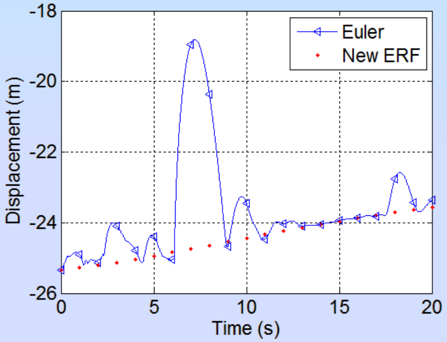


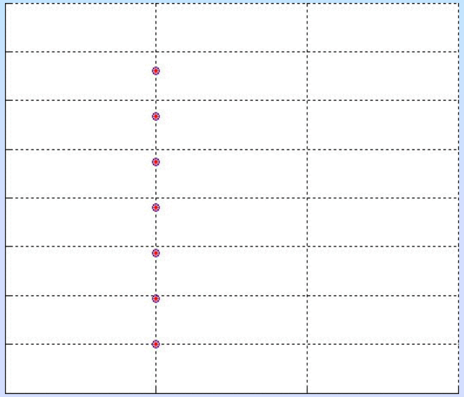


Move **Y**-directionally with **Y**-directional current







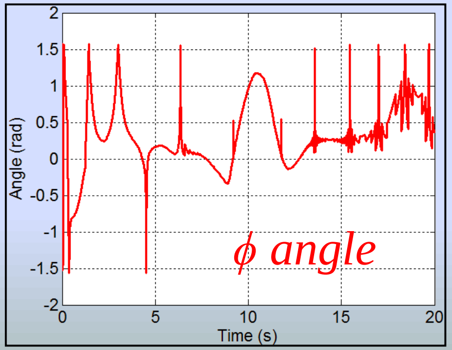


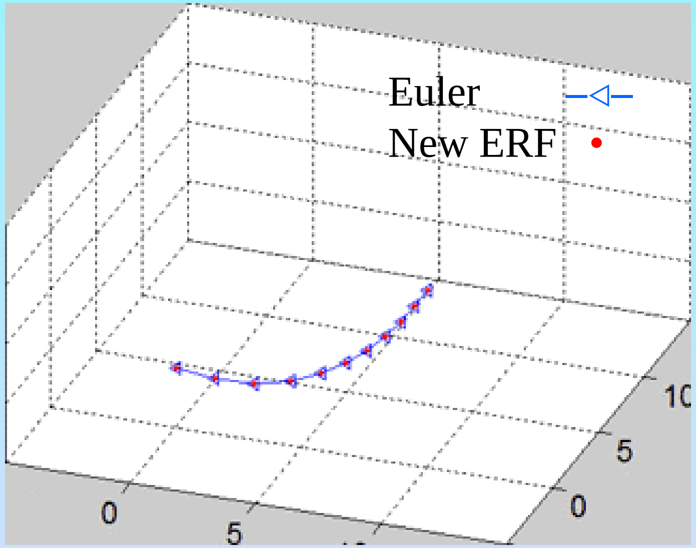
$$\theta^i = a\tan 2\left(E_g^{i,1}, E_g^{i,3}\right)$$

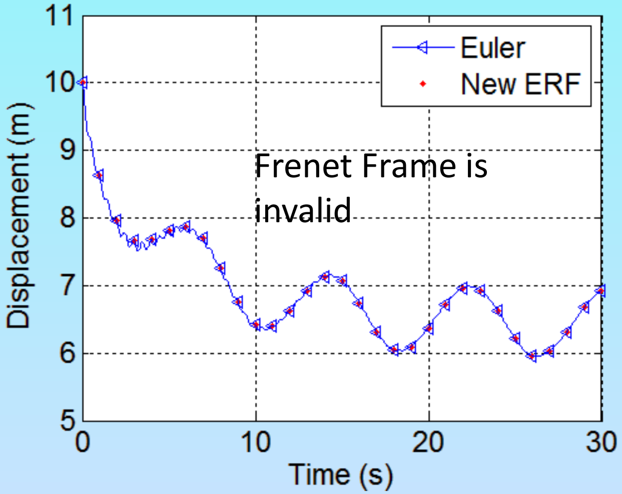
$$\phi^i = a\tan 2\left(-E_g^{i,2}, \frac{E_g^{i,3}}{\cos\theta^i}\right), \quad if \quad \cos\theta^i > \sin\theta^i$$

$$\phi^i = a\tan 2\left(-E_g^{i,2}, \frac{E_g^{i,1}}{\sin\theta^i}\right), \quad if \quad \cos\theta^i < \sin\theta^i$$

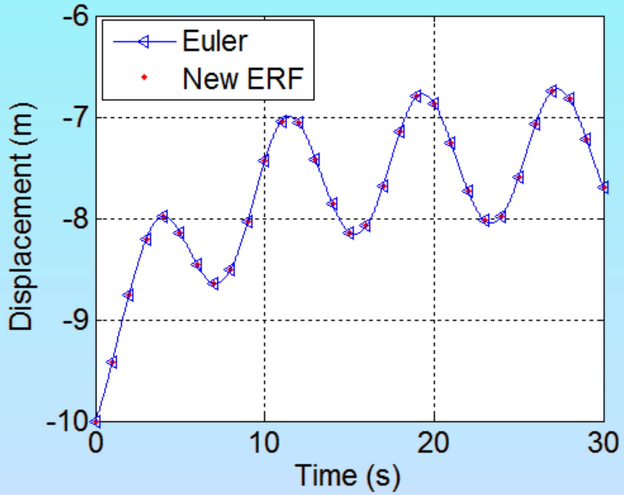
X-directional difference is 0, so the rotation angle θ is 180°, which results in the denominator of the atant2 function being 0. This forces the rotation angle ϕ being pi/2 or –pi/2 easily in spite of the values in the front component of the atan2 function. Simulation results indicate that the rotation angle ϕ is pi/2 or –pi/2 interactively



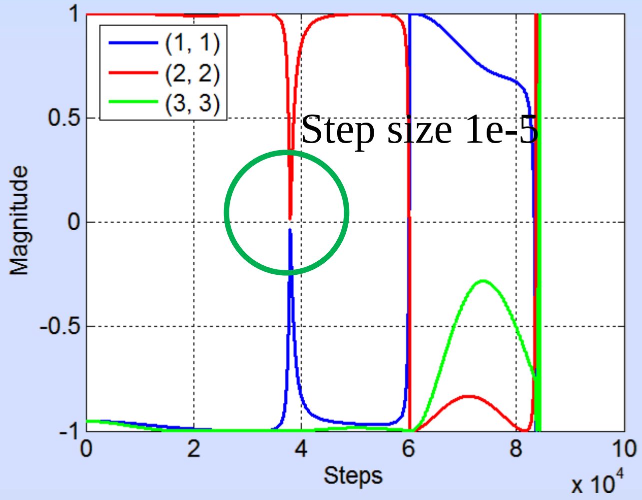




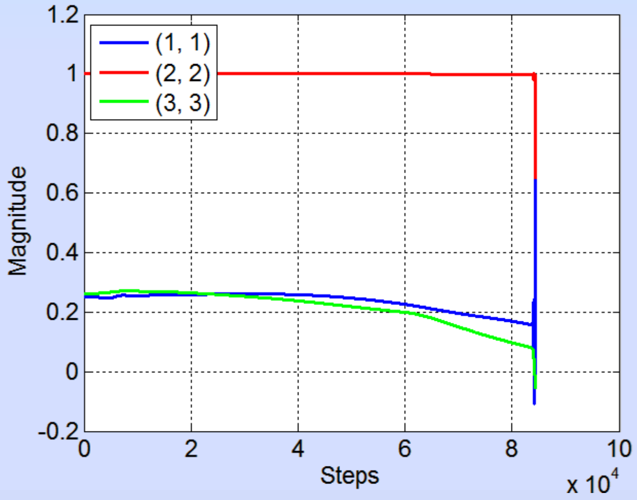
X-disp. of the 1st node



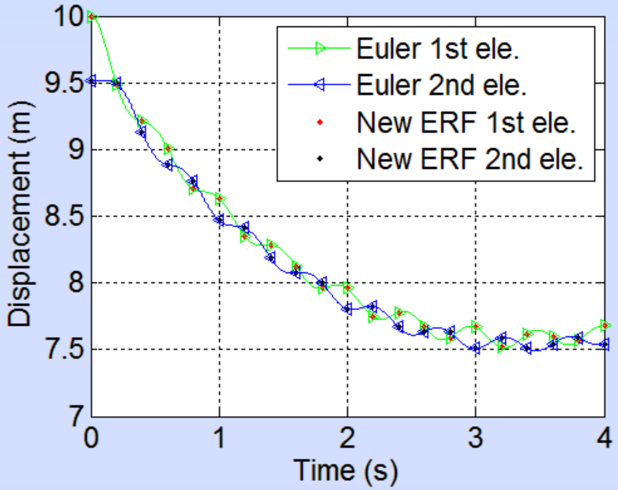
Z-disp. of the 1st node



Components of the RTM by Frenet Frame in singularity cases(the 1st element)

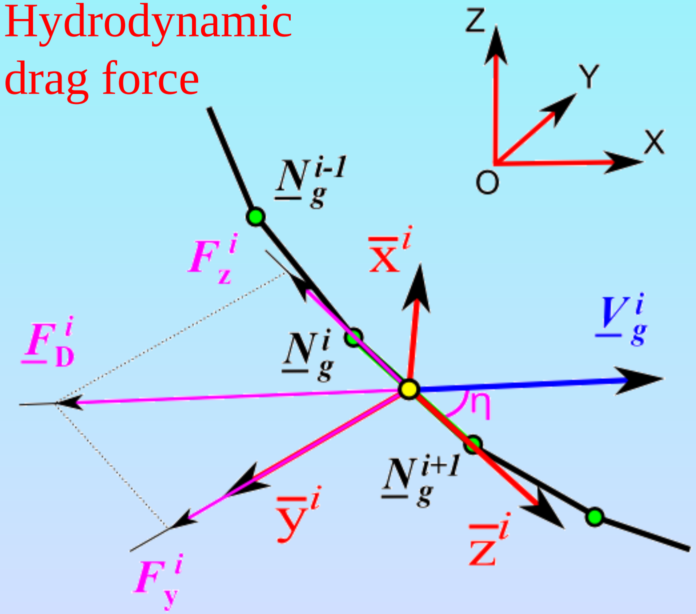


Components of the RTM by Frenet Frame in singularity cases(the 10th element)



X-disp. of the 1st and 2nd nodes

Hydrodynamic drag force



Frenet frame

Relative velocity in ERF from the IRF

⇓

3 drag forces in element frame with relative velocity in ERF

⇓

3 drag forces in IRF

New element frame

2 drag forces in element frame with relative velocity in IRF

⇓

3 drag forces in IRF

NB

$$F_x^i = -\frac{1}{2} C_n \rho_f d^i l^i \left|V_b^{R,x}\right| V_b^{R,x}$$

$$F_y^i = -\frac{1}{2} C_n \rho_f d^i l^i \left|V_b^{R,y}\right| V_b^{R,y}$$

$$F_z^i = -\frac{\pi}{2} C_f \rho_f d^i l^i \left|V_b^{R,z}\right| V_b^{R,z}$$

NG

$$F_y^i = \frac{1}{2} C_n \rho_f d^i l^i \cdot \left\| \tilde{z} V_g^i \right\|^2$$

$$F_z^i = -\frac{\pi}{2} C_f \rho_f d^i l^i \cdot \bar{z}^T V_g^i \cdot \left|\bar{z}^T V_g^i\right|$$

NBL

$$F_x^i = -\frac{1}{2} C_n \rho_f d^i l^i f_p \left|V_b^R\right| \frac{V_b^{R,x}}{\sqrt{\left(V_b^{R,x}\right)^2 + \left(V_b^{R,y}\right)^2}}$$

$$F_y^i = -\frac{1}{2} C_n \rho_f d^i l^i f_p \left|V_b^R\right| \frac{V_b^{R,y}}{\sqrt{\left(V_b^{R,x}\right)^2 + \left(V_b^{R,y}\right)^2}}$$

$$F_z^i = -\mathrm{sgn}(V_b^{R,z}) \frac{1}{2} C_n \rho_f d^i l^i f_q \left|V_b^R\right|^2$$

Relationship

Relative Velocity & Unit-axes of frame

⇓

$$V_b^{i,y} = -\left|V_g^i\right| \sin\eta$$

$$V_b^{i,z} = \left|V_g^i\right| \cos\eta$$

$$V_b^{i,y} = -\left\| \tilde{z} V_g^i \right\|$$

$$V_b^{i,z} = \bar{z}^T V_g^i$$

Empirical formulation with

Loading function parameters

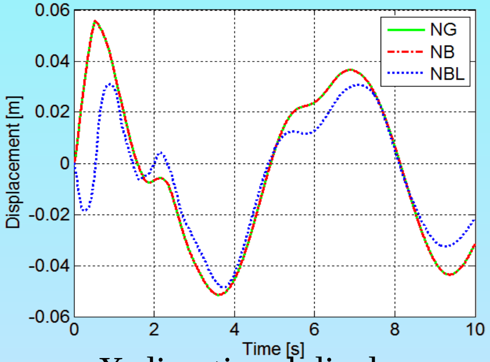
where

$$f_p = 0.5 - 0.1\cos(\eta) + 0.1\sin(\eta) - 0.4\cos(2\eta) - 0.11\sin(2\eta)$$

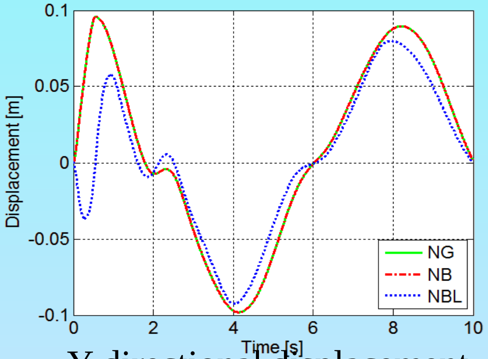
$$f_q = 0.01\left(2.008 - 0.3858\eta + 1.9159\eta^2 - 4.1615\eta^3 + 3.5064\eta^4 - 1.1873\eta^5\right)$$

$$\eta = a\cos\frac{V_b^{R'} \cdot \bar{z}}{\left|V_b^R\right| \left|\bar{z}\right|} \qquad \left(0 \le \eta \le \frac{\pi}{2}\right)$$

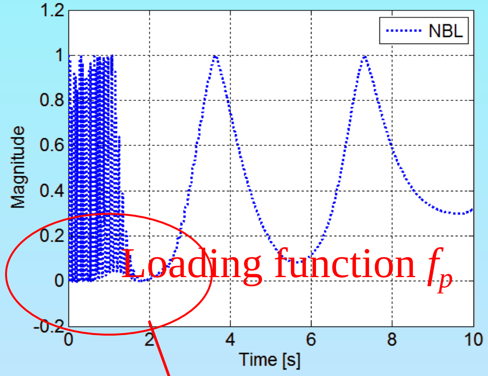
CABLE MODELING---DRAG FORCE

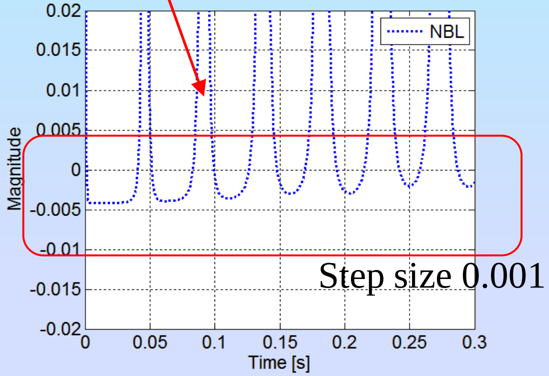


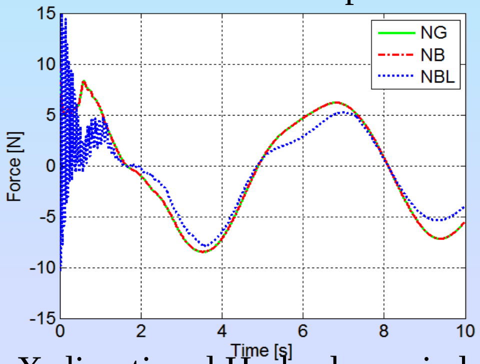
X-directional displacement



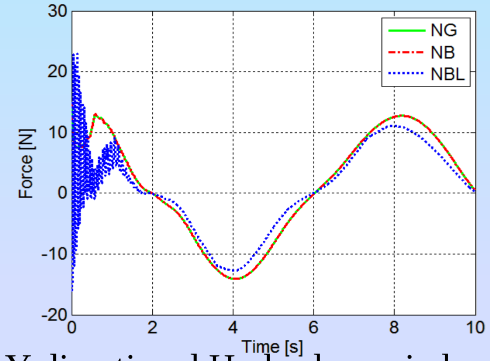
Y-directional displacement



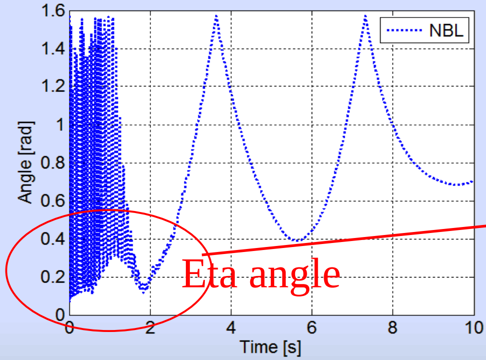


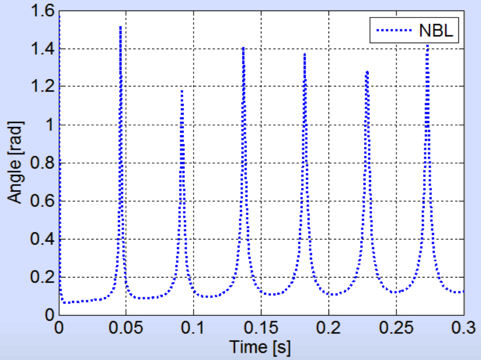


X-directional Hydrodynamic loads



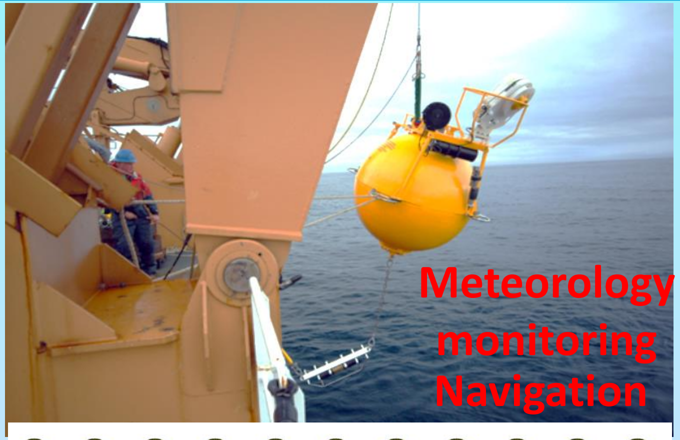
Y-directional Hydrodynamic loads

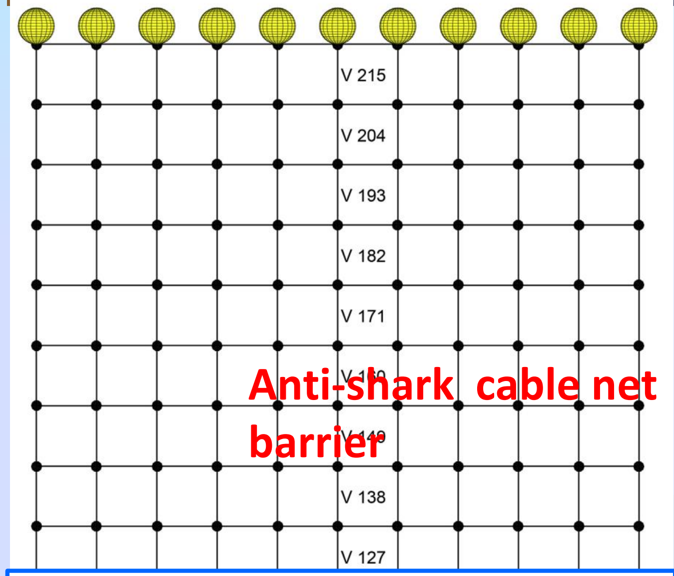




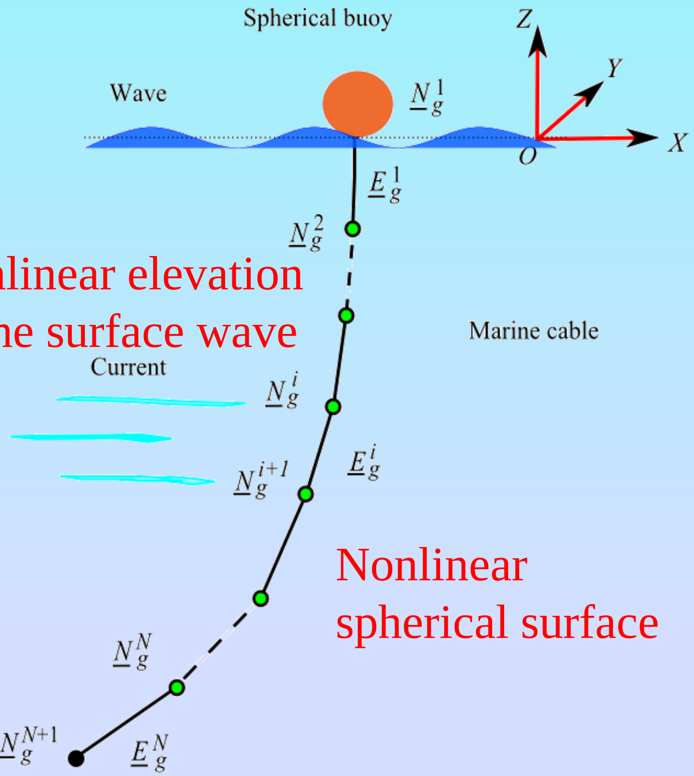
The f_p is negative when the ***Eta*** is close to the 0.1, which changes the direction of the hydrodynamic drag force. Therefore, the hydrodynamic drag force based on the loading functions is incorrect when the cable is pulled taut. The formulation based on the new element frame is appropriate in the case of vertically taut cables.

Zhu Xiang Qian, Yoo Wan Suk. Suggested New Element Reference Frame for Dynamic Analysis of Marine Cables; **Nonlinear Dynamics**; 87(1), p489–501; 2017.*





The size of the spherical buoy is small, so the dynamics of the cable have obvious influence on the motions of the system.



Nonlinear elevation of the surface wave

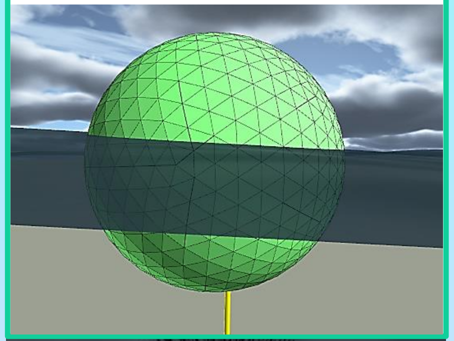
Nonlinear spherical surface

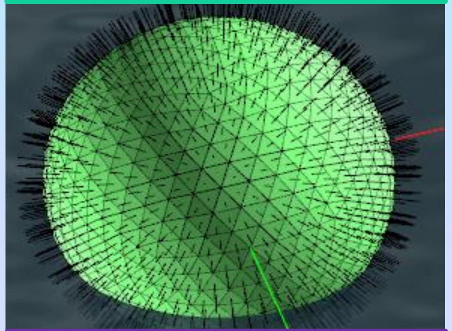
Efficient modeling is preferred !

Assumptions

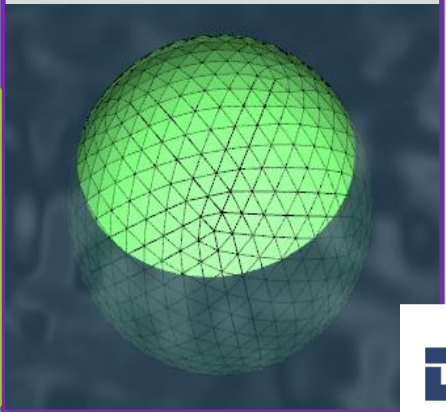
1. Closed curve is co-plane.
2. Morison equation is valid.
3. Rotational motions of the buoy are ignored.
4. All the loads acting on the 1st node of the cable.

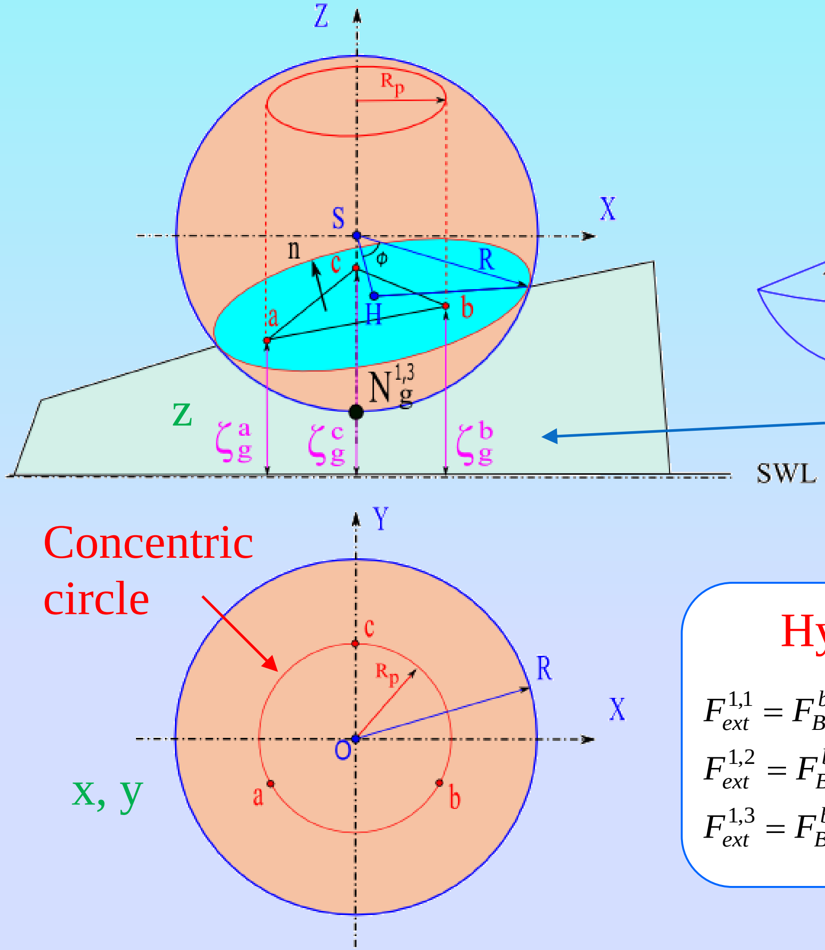
Polyhedral mesh

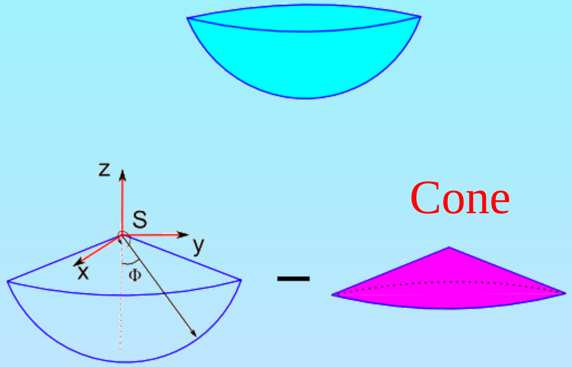




Closed curve







$$\zeta = \zeta_a^x \cos(k^x X_g - \omega^x t) + \zeta_a^y \cos(k^y Y_g - \omega^y t)$$

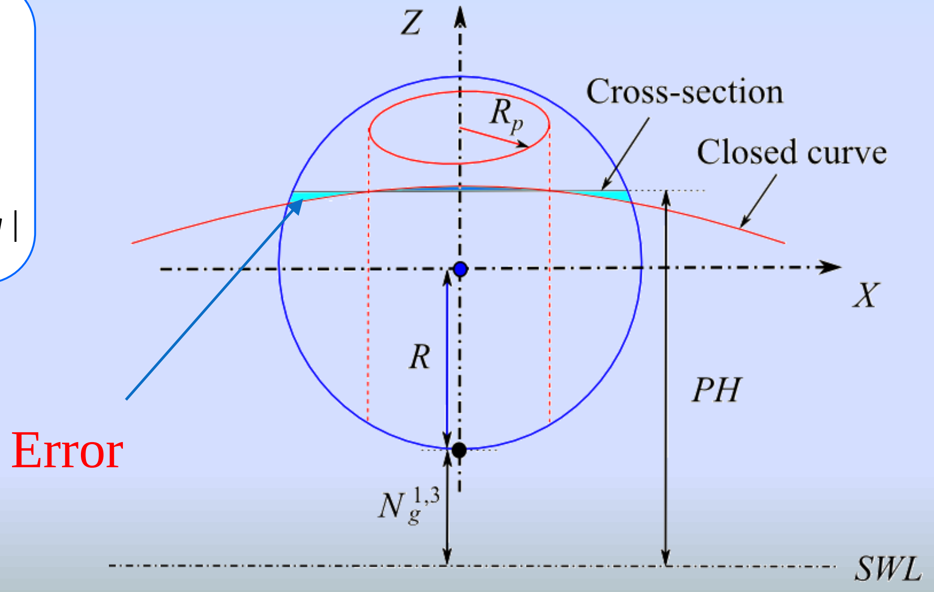
Submerged Volume

$$V_s = \begin{cases} \int_0^{2\pi} d\theta \int_0^{\phi} \sin\phi \, d\phi \int_0^{R} r^2 dr - \frac{\pi}{3} SH(R^2 - SH^2) \\ if \quad PH < N_g^{1,3} + R \\ \frac{4}{3}\pi R^3 - \int_0^{2\pi} d\theta \int_0^{\phi} \sin\phi \, d\phi \int_0^{R} r^2 dr + \frac{\pi}{3} SH(R^2 - SH^2) \\ if \quad PH \geq N_g^{1,3} + R \end{cases}$$

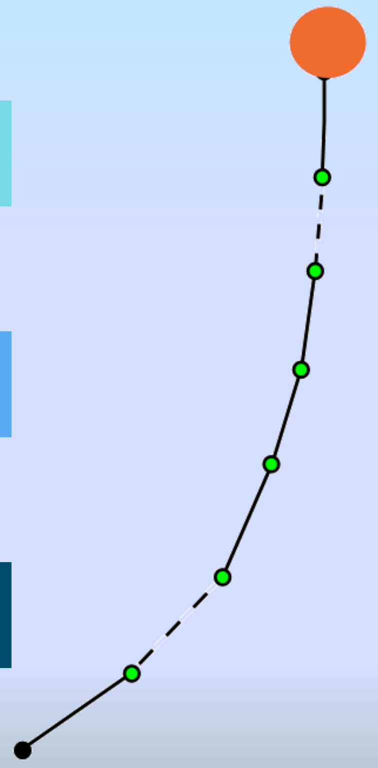
Hydrodynamic loads are expressed on the 1st node of the cable based on the submerged volume

Hydrodynamic Loads

$$F_{ext}^{1,1} = F_B^{buoy,1} + F_A^{buoy,1} + F_D^{buoy,1}$$
$$F_{ext}^{1,2} = F_B^{buoy,2} + F_A^{buoy,2} + F_D^{buoy,2}$$
$$F_{ext}^{1,3} = F_B^{buoy,3} + F_A^{buoy,3} + F_D^{buoy,3} - m^{buoy} |g|$$



The cross-section is determined by three points, a, b and c, which locate on the edge of an assumed concentric circle.



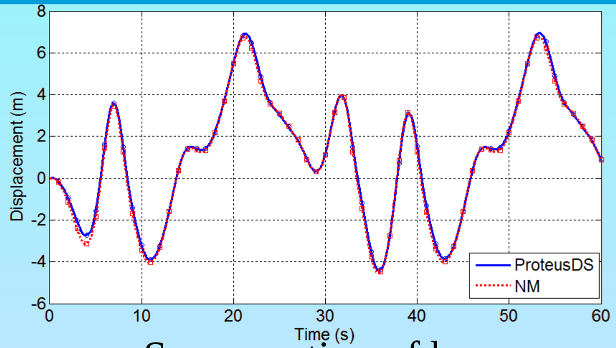
Cable Property Parameters	Magnitude	Unit
Diameter	0.03	m
Density	1570	kg/m^3
Elastic modulus	2.38e9	N/m
Damping coefficient	1.0e4	Ns/m
Normal drag coefficient	1	
Tangential drag coefficient	0.01	
Added mass coefficient	0.5	
Position of 1^{st} node	[10; 0; -10]	m
Position of 11^{th} node	[0; 0; -30]	m

Spherical Buoy Parameters	Magnitude	Unit
Radius	1	m
Mass	500	kg
Radius of the concentric circle	0.8	m
Drag coefficient	1	
Added mass coefficient	1	

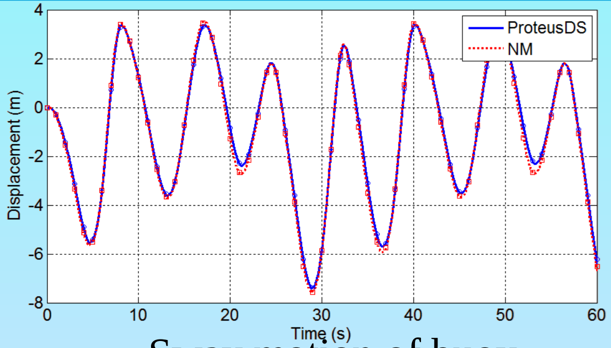
Ocean State Parameters	Magnitude	Unit
X-directional wave amplitude	0.7	m
Y-directional wave amplitude	6.4	m
X-directional wave period	1.2	s
Y-directional wave period	8	s
Velocity of current	[1; 0; 0]	m/s
Density of fluid	1025	kg/m^3
Stiffness of seabed	87	MN/m
Damping of seabed	33	MN • s/m

Efficiency Comparison

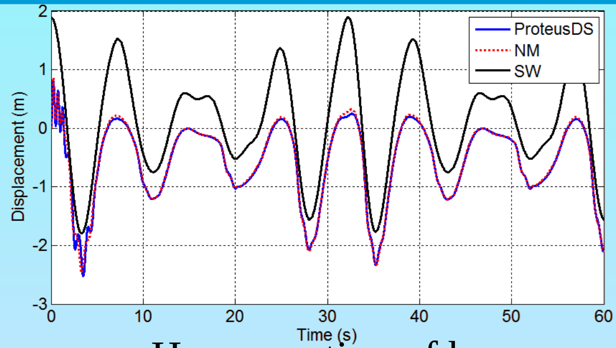
Parameters	ProteusDS	NM Code
Simulation time	20 s	20 s
Integrator	RK 4	RK 4
Step size	1e-4	1e-4
Real time	2655 s	1699 s
Rate	1	64 %



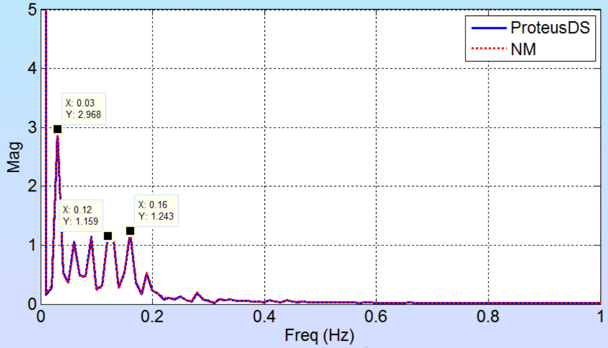
Surge motion of buoy



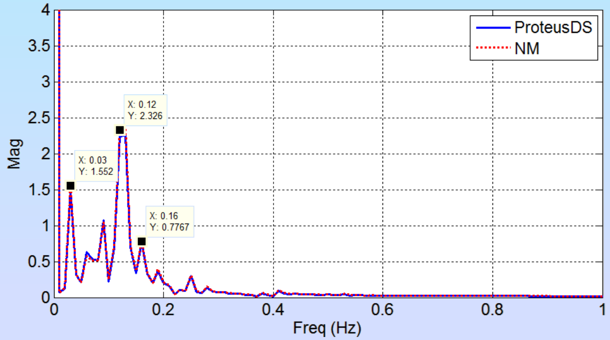
Sway motion of buoy



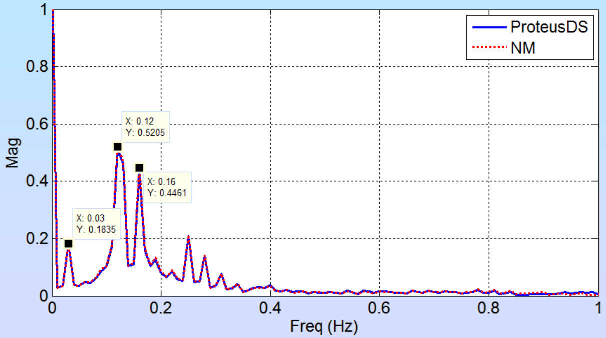
Heave motion of buoy



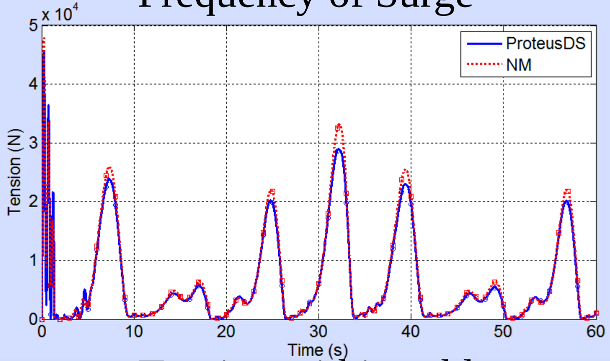
Frequency of Surge



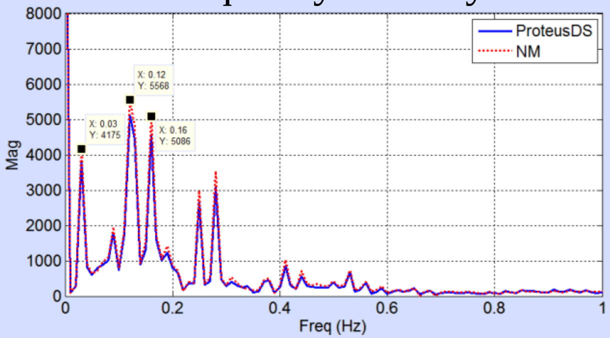
Frequency of Sway



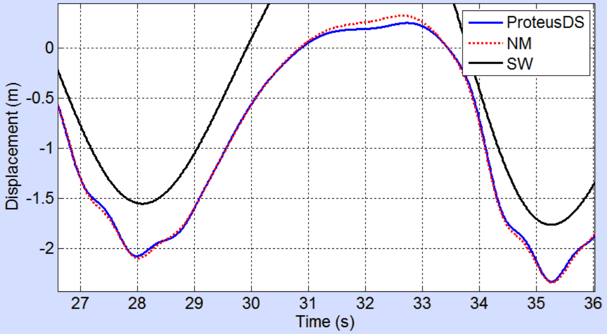
Frequency of Heave



Tension within cable



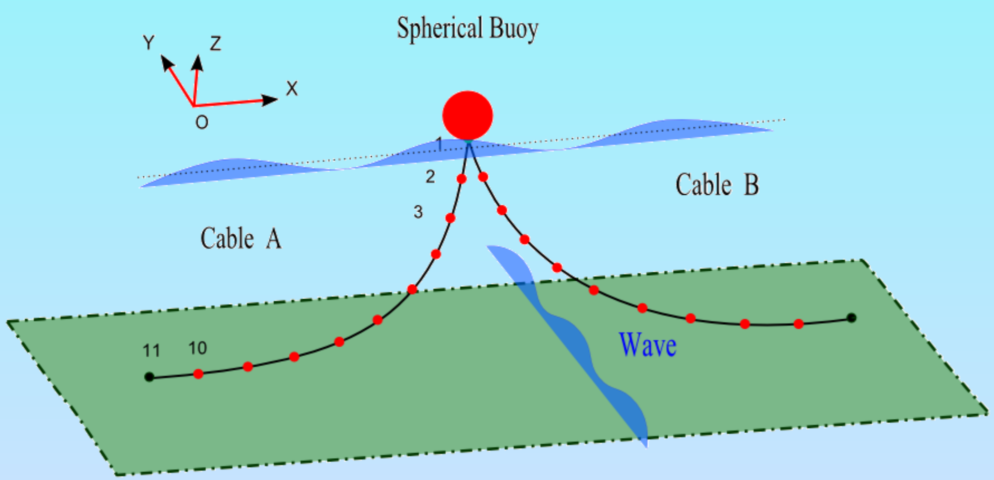
Frequency of Tension

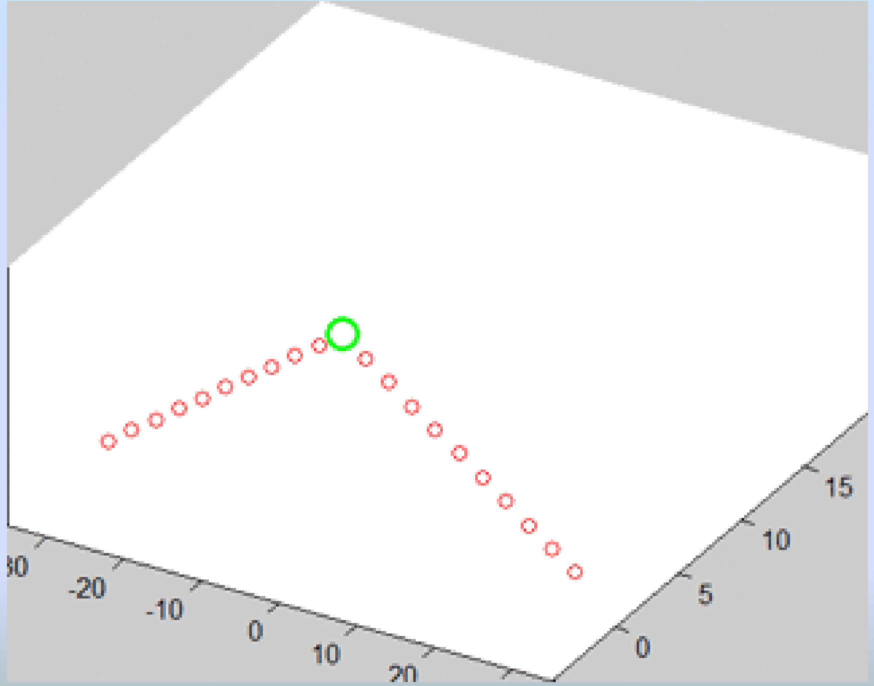


Enlarged heave motion

Zhu Xiang Qian, Yoo Wan Suk. **Numerical modeling of a spherical buoy moored by a cable in three dimensions**; Chinese Journal of Mechanical Engineering; 29(3);p588-597; 2016.*

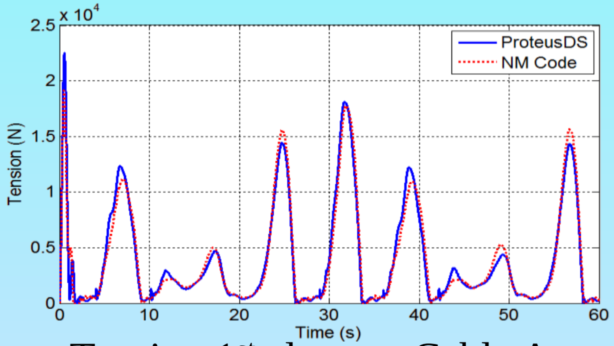
SPHERICAL BUOY



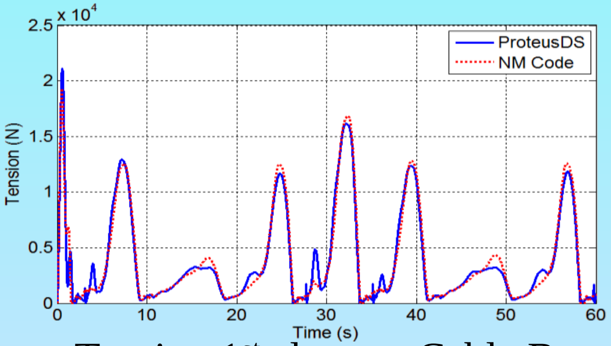


Ocean State Parameters	Magnitude	Unit
X-directional wave amplitude	0.7	m
Y-directional wave amplitude	1.2	m
X-directional wave period	6.4	s
Y-directional wave period	8	s

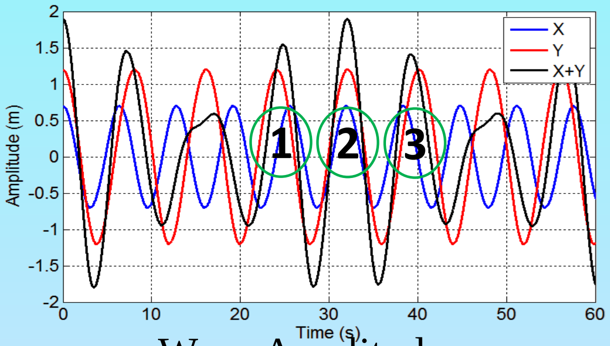
Cable Property Parameters	Magnitude	Unit
Diameter	0.03	m
Density	3570	kg/m^3
Elastic modulus	2.38e9	N/m
Damping coefficient	1.0e4	Ns/m
Normal drag coefficient	1	
Tangential drag coefficient	0.01	
Added mass coefficient	0.5	
Position of the top node	[0; 0; 0]	m
Bottom node of cable A	[-30; 0; -20]	m
Bottom node of cable B	[30; 0; -20]	m



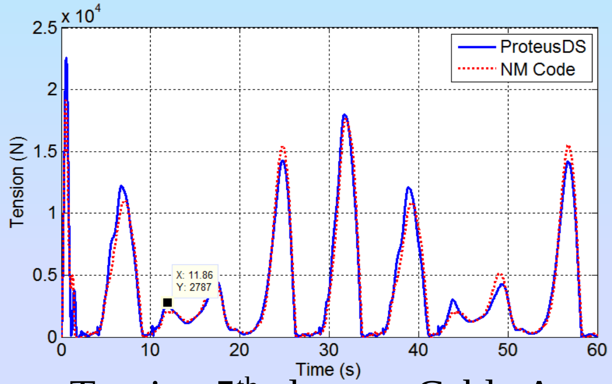
Tension 1st element Cable A



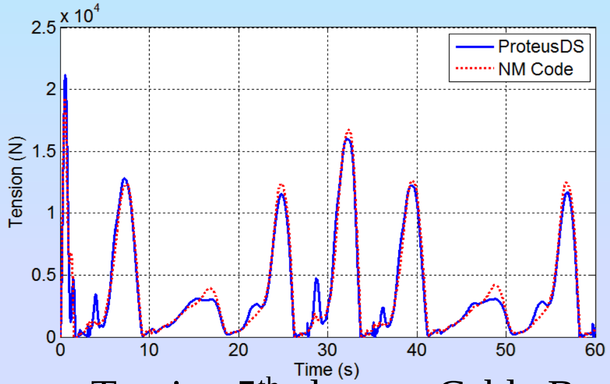
Tension 1st element Cable B



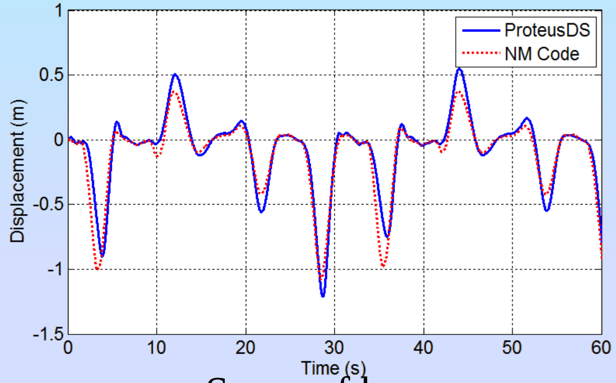
Wave Amplitude



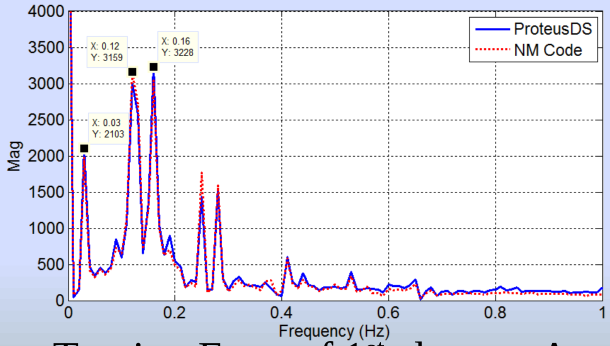
Tension 5th element Cable A



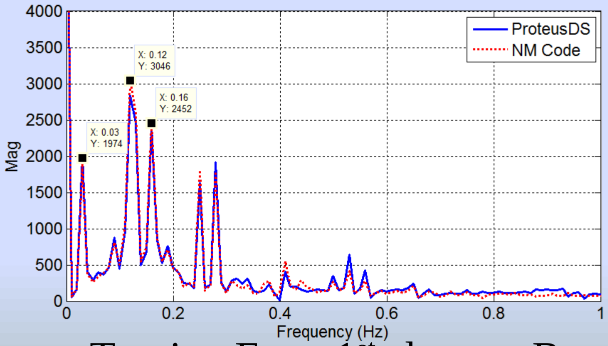
Tension 5th element Cable B



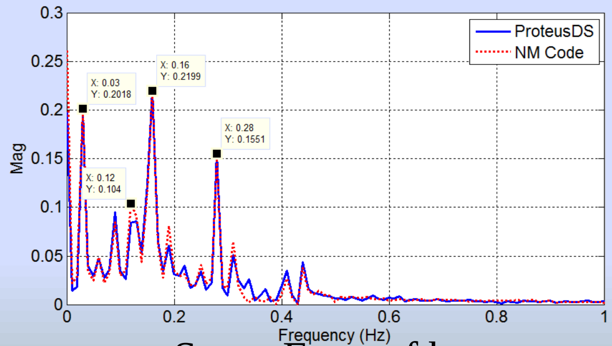
Surge of buoy



Tension Freq. of 1st element A

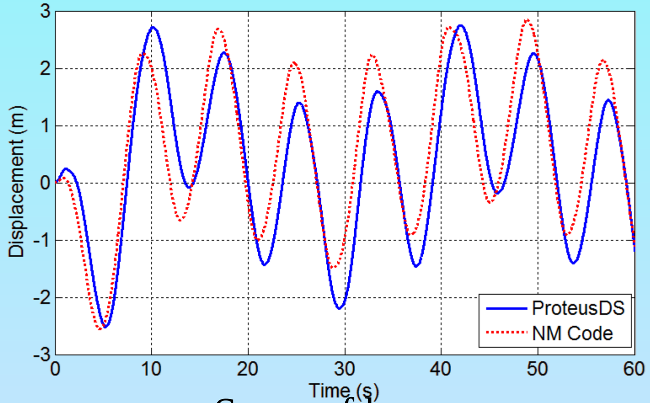


Tension Freq. 1st element B

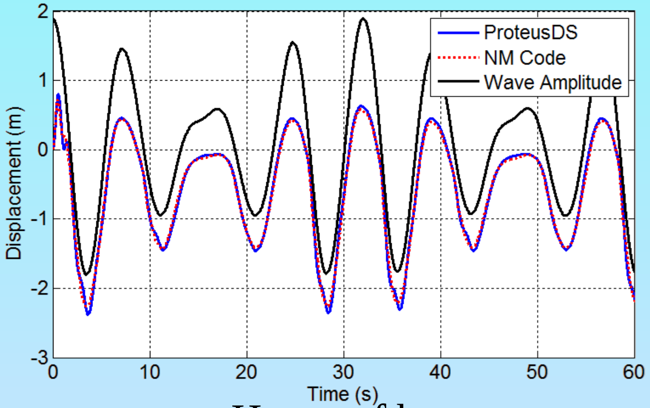


Surge Freq. of buoy

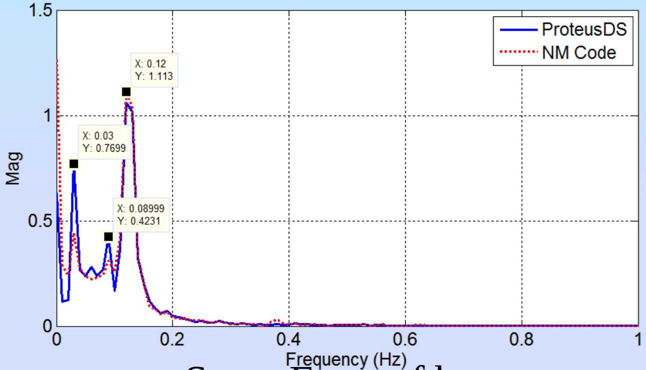
SPHERICAL BUOY



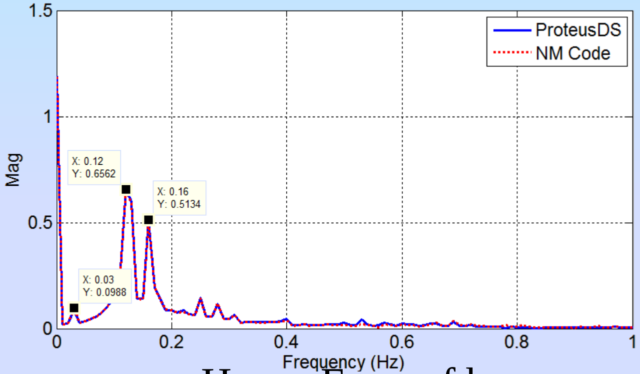
Sway of buoy



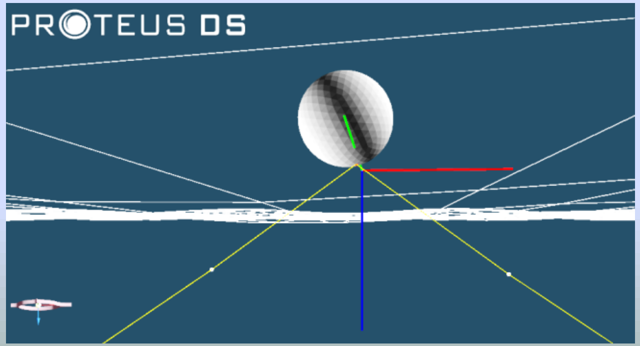
Heave of buoy

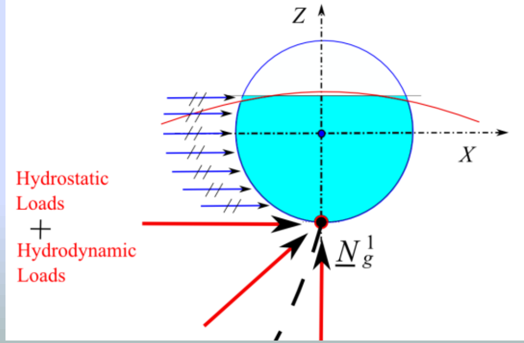


Sway Freq. of buoy



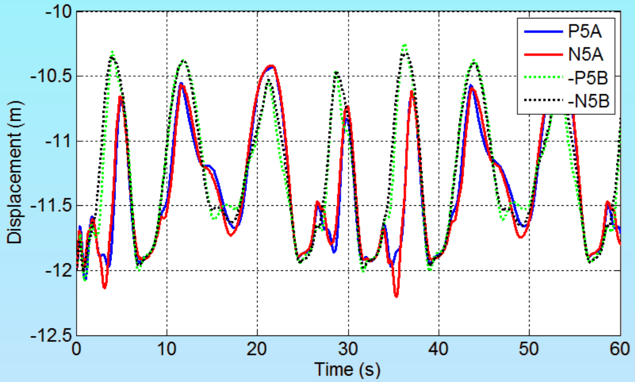
Heave Freq. of buoy



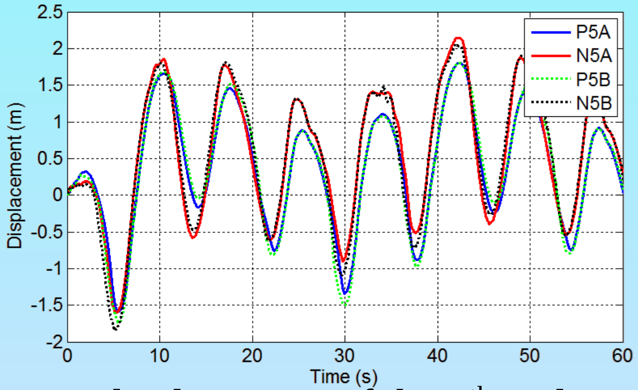


The Roll and Pitch motions of the buoy is ignored in the numerical modeling. It assume the ligature between the center of the buoy and the node is vertical upward during the simulation. While, the buoy has the rotational motions in ProteusDS, so the buoy can avoid the wave crest by a short time. This phenomenon enable a time delay to exist in the motion of the buoy.

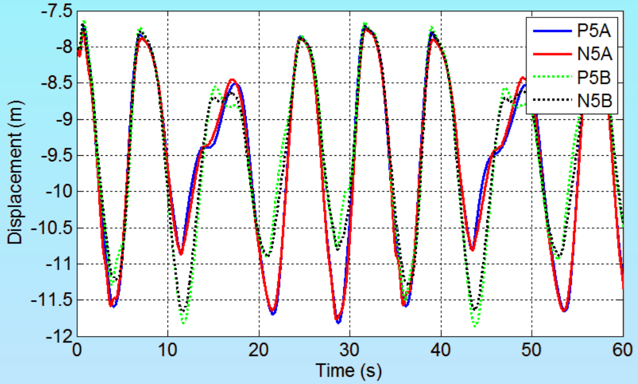
All the loads acting on the buoy, including the Hydrostatic and Hydrodynamic loads, expressed on the bottom of the buoy, which is the 1st node of the cable in the NM Code. The hydrodynamic loads obtained by the proposed modeling are smaller than those obtained by the Polygonal mesh.



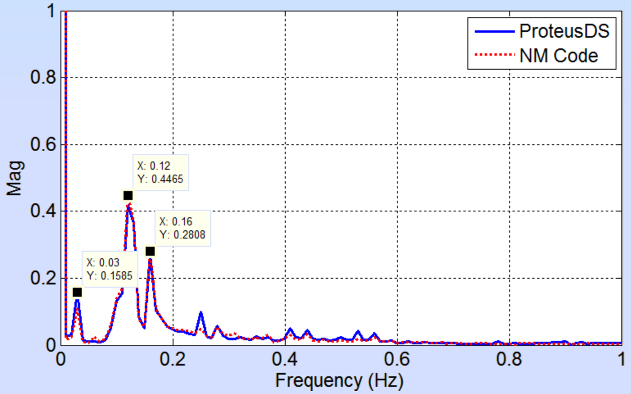
X-displacement of the 5th nodes in Cable A and B



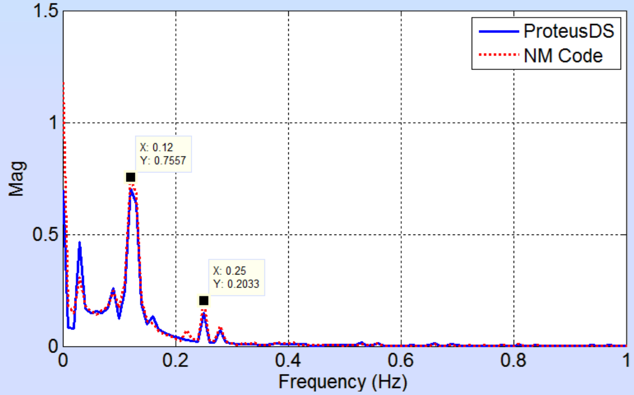
Y-displacement of the 5th nodes in Cable A and B



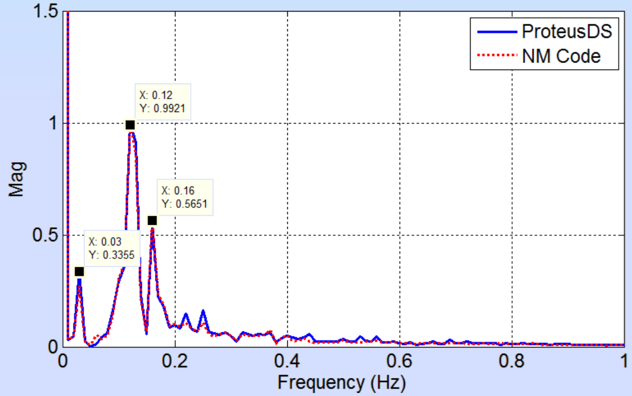
Z-displacement of the 5th nodes in Cable A and B



Freq. of x-displacement of the 5th nodes in Cable B



Freq. of y-displacement of the 5th nodes in Cable B



Freq. of z-displacement of the 5th nodes in Cable B

Zhu Xiang Qian, Yoo Wan Suk. **Dynamic Analysis of a Floating Spherical Buoy Fastened by Mooring Cables**; *Ocean Engineering; 121, p462–471. 2016;*

Summary

- By means of the relative velocity, a new element reference frame is constructed for modeling marine cable.
- The hydrodynamic loads are expressed efficiently, but also many singular problems can be overcome easily.
- By means of the special coordinate, a analytical method is proposed to characterize the hydrodynamics acting on the spherical buoy.
- The relationships between the motions of the system and propagating waves are studied, and the simulation results are verified by commercial software ProteusDS.

Acknowledgement:
The DSA Ltd. for providing the software ProteusDS

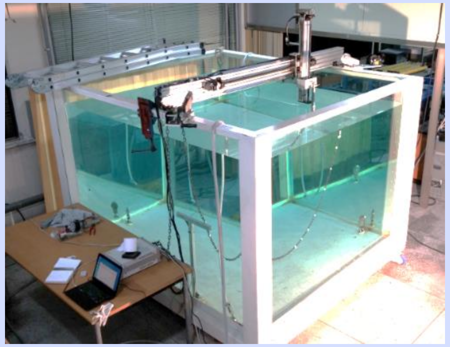
Other Publications

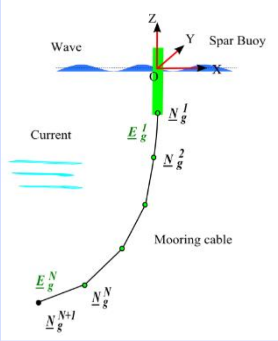
Zhu Xiang Qian，Yoo Wan Suk.* ***Verification of a Numerical Simulation Code for Underwater Chain Mooring****；Archive of Mechanical Engineering；63（2）；p231-244; 2016*

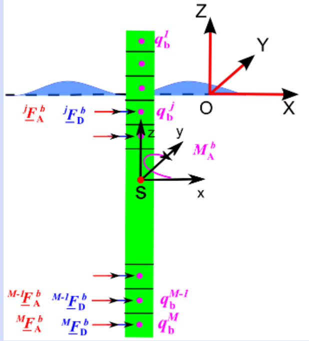
Zhu Xiang Qian，Yoo Wan Suk.****Numerical Modeling of a Spar Platform Tethered by a Mooring Cable****；Chinese Journal of Mechanical Engineering；28(4)，p785–792；2015.*

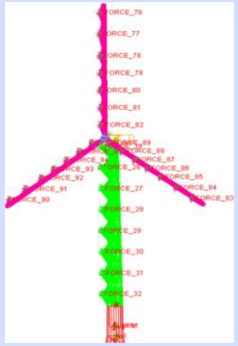
Zhu Xiang Qian，Yoo Wan Suk.* ***Flexible dynamic analysis of an offshore wind turbine installed on a floating spar platform****；Advances in Mechanical Engineering；8（6）；p1-11; 2016*

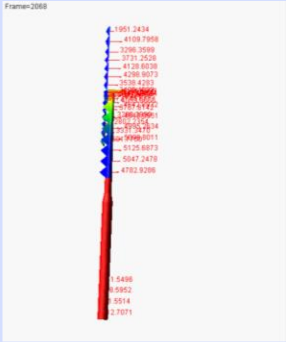
Jiang Zhiyu, Zhu Xiang Qian, Hu weifei.* ***Modeling and Analysis of Offshore Floating Wind Turbines****, Chapter 9,《Advanced Wind Turbine Technology》, Springer International Publishing, 2018*

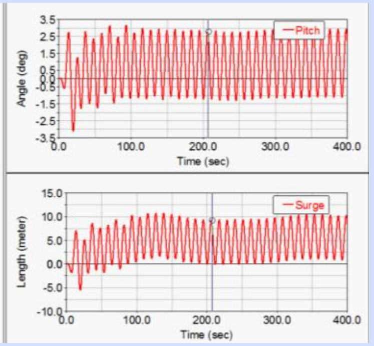












Thank you very much!